formation of such a configuration of explosions comes about without reference to whether or not the second explosion (i.e., the shock front) overtakes the first explosion on the asymptote or not. The amplitude of the excess pressure in the first SW at distances of r_1' from the axis corresponds approximately to 1.0, i.e., at distances of $r' > r_1'$ the evolution of the double wave truly corresponds to the quasiacoustic stage.

Within the scope of approximating nonlinear acoustics it is demonstrated analytically in [2] that the formation of double SW configurations with constant time spacing T = const between the fronts is associated with the specific agreement of the amplitudes and profiles of the two waves at the point $r' = r_1'$.

Thus, we can see from these calculations that for a cylindrical double explosion, as well as for a spherical explosion, there exists a region of control-parameter values t_0, λ in which the second wave, within the period of evolution from $r' = 0$ to $r' = r_1'$, "positions" itself with respect to the first wave in a manner such that it becomes possible to form two-wave configurations with quasiconstant spacing T between the fronts. The spacing T and the interval $\Delta r' = r_2' - r_1'$ depend exclusively on λ and t_0 and can be determined only as a result of a numerical solution for the problem of a double explosion.

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MICROSCOPIC CONDITIONS FOR THE EXISTENCE OF RAREFACTION SHOCK WAVES IN SOLIDS

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The existence of rarefaction shock waves in a substance near the critical point of the Ist kind of phase transition was predicted by Zel'dovich [1] and observed experimentally [2-4].

Let us examine the conditions for the existence of rarefaction shock waves, such as are associated with phase transition of the IInd kind.

In some manner let us initiate a multilateral rarefaction wave of amplitude P in a material subjected to preliminary stress, said wave of rather limited width such that the time required for a change in pressure is smaller than the stress relaxation time within the material. It is assumed that the body in the solid state with expansion such that $\Delta V = V_L - V_m$ (V_L, V_m is the volume of the body in the liquid and solid phases, respectively) makes the transition to the metastable state [5]. In this case, if the body remains in the solid phase, the new state may be amorphous [6]. With such transitions the body undergoes continuous changes of state, whereas the symmetry undergoes sudden jumps. We know that an amorphous structure is, in and of itself, more symmetrical than any ordered structure. The process involved in the formation of a new structure under the action of a rarefaction wave proceeds through a series of intermediate structures whose crystallographic symmetry covers more than 230 spatial groups [7], i.e., the transition process is represented by a sequence of states with ever-broader classes of symmetry. The conclusion of this process is found in the transition of the material into a fully amorphous state (a phase transition of the IInd kind [8]).

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From the Chapman–Jouguet equation

$$S_{II} - S_I = \frac{1}{12}\frac{1}{T}\left(\frac{\partial^2 V}{\partial p^2}\right)_S (p_{II} - p_I)^3$$

(p, pressure; S, entropy; T, temperature; subscripts I and II correspond to states I and II) follows the condition for the existence of a shock (sharp) rarefaction wave [1]

$$(\partial^2 V/\partial p^2)_S < 0. \tag{1}$$

Let us examine the possibility of satisfying conditions (1) in the transition to a noncrystalline state under the action of a rarefaction wave. We will write the thermodynamic potential of the system as a function of the pressure p, the temperature, and field strength h: $\Phi = \Phi(p, h, T)$. The potential Φ is represented in the form [8]

$$\Phi(p, T, h) = \Phi_0(p, T) + at^2\eta^2 + B\eta^4 - \eta hV, \tag{2}$$

where η is the parameter of the order on which h is dependent; $t = t - T_c(p)$; $T_c(p)$ is the temperature of the phase transition.

The equilibrium condition $(\partial\Phi/\partial\eta)_{T,h} = 0$ with consideration of (2)

$$2at^2\eta + 4B\eta^3 = hV. \tag{3}$$

If the curve of (3) is nonmonotonic, it will clearly exhibit a bending point. Let us examine the case in which the curve is monotonic. The transition to a more symmetric phase must be accompanied by a drop in temperature and therefore $t < 0$, while for the sake of monotonicity it is necessary that $at > 0$, so that consequently $a < 0$ [8]. Such a selection of the parameter imposes no limitations on the generality of our examination. The change of the intensity h to 0 corresponds to the situation in which the potential in (2), $\Phi(p, T, h) = \Phi(p, T)$ and the existence of a bend in the potential are equivalent to the existence of a point of bending in the T(p) curve. This gives us

$$-(dV/dp)_S = C_V/T_c(dT_c/dp)^2$$

(C_V is the heat capacity with a constant volume).

The following relationship is valid on the phase-transition curve:

$$(\partial p/\partial T)_V = dp_c/dT$$

(p_c is the pressure, and the subscript c here and below indicates that the quantities have been taken from the transition curve). If we bear in mind that $T = T(p)$, and if we differentiate with respect to p, we can write

$$-\left(\frac{\partial^2 V}{\partial p^2}\right) = \frac{C_V}{T_c^2}\frac{dT_c}{dp}\left(\frac{dT_c}{dp}\right)^2 + \frac{2C_V}{T_c}\frac{dT_c}{dp}\frac{d^2T_c}{dp^2} = \frac{C_V}{T_c}\frac{dT_c}{dp}\left(2\frac{d^2T_c}{dp^2} - \frac{1}{T_c}\left(\frac{dT_c}{dp}\right)\right)^2.$$

Because of the fact that $T_c(p)$ is a monotonic function, the sign of its derivative dT_c/dp does not change. We can always require near the point of the bend that d^2T_c/dp^2 be less (or greater) than $(1/2T_c)(dT_c/dp)^2$, and consequently, the condition for the existence of the rarefaction shock wave (1), associated with the transition to the noncrystalline metastable state, can be satisfied.

The conditions

$$\frac{d^2T_c}{dp^2} > \frac{1}{2T_c}\left(\frac{dT_c}{dp}\right)^2; \tag{4}$$

$$\frac{d^2T_c}{dp^2} < \frac{1}{2T_c}\left(\frac{dT_c}{dp}\right)^2 \tag{5}$$

determine the required shape of the curve for phase equilibrium between the crystalline metal and the amorphous metal and impose limitations on the material in which these rarefaction shock waves can be initiated. The choice of one of the inequalities (4) and (5) depends on the sign of the derivative dT_c/dp. Determining the sign at the point of transition is a particular problem. Proceeding from the microscopic theory of metals [9], we can write

$$K = (0.0275 + 0.1102 k_F r_c^2) k_F^5 \cdot 10^{12}, \tag{6}$$

where K is the isothermal modulus of multilateral compression; k_F is the Fermi wave vector; r_c is the equilibrium distance between the atoms. In Eq. (6) $[k_F] = Å^{-1}$, $[r_c] = Å$, and thus K is expressed in units of J/m^3. With consideration of the expressions for the Fermi vector $k_F = (3\pi^2 z/\Omega_0)^{1/3}$ Eq. (6) is rewritten to the form

$$K = \frac{C_1 z^{5/3}}{(\Omega_0 (1 + \alpha \delta T))^{5/3}} + \frac{C_2 z^2 r_0^2 \left(1 + \frac{2}{3} \delta T\right)^2}{\Omega_0^2 (1 + \alpha \delta T)^2}. \tag{7}$$

Here $C_1 = 0.0275 \cdot 10^{12} (3 \cdot \pi)^{5/3}$, $C_2 = 0.1102 \cdot 10^{12} (3 \cdot \pi)^2$ are the numerical constants; Ω_0 is the volume per single atom; z represents the number of valence electrons. Provision has been made in (7) for the thermal expansion of the metal $r_c = r[1 + (\alpha/3)\delta T]$ and $\Omega = \Omega_0(1 + \alpha \delta T)$ (α is the coefficient of thermal expansion, δT represents the change in temperature within the shock wave). Differentiating K with respect to T, we have

$$\frac{dK}{dT} = \left(\left(\frac{C_1 z}{\Omega_0} \right)^{5/3} \frac{\delta T}{(1 + \alpha \delta T)^{8/3}} + \frac{C_2 z^2 r_0^2}{\Omega_0} \left(-\frac{4}{3} \right) \frac{1}{(1 + \alpha \delta T)^2} \right) \frac{d\alpha}{dT}. \tag{8}$$

We know that

$$\beta = \frac{1}{p_0} \left(\frac{\partial p}{\partial T} \right)_V = \frac{V_0}{p_0 V} \alpha K \tag{9}$$

(β is the thermal coefficient of pressure and V_0, p_0 represents the initial volume and pressure). Then

$$\frac{\partial \beta}{\partial T} = \frac{1}{p_0} \frac{\partial^2 p}{\partial T^2} = \frac{1}{p_0} \frac{d^2 p_c}{dT_c^2} = \frac{1}{p} \left(\frac{V_0 \alpha^2}{p_0 V^2} K + \frac{V_0}{p_0 V} \alpha \frac{dK}{dT_c} + \frac{V_0}{p_0} K \frac{d\alpha}{dT_0} \right). \tag{10}$$

The transition from partial to total derivatives in (10) is based on the fact that the derivative is taken from the phase-transition curve. We cannot neglect $d\alpha/dt$ in (10), since its magnitude is substantial on the phase-transition curve.

For the sake of specificity we will examine condition (4). We will rewrite it as follows:

$$\frac{1}{d^2 p / dT_c^2} > \frac{2T_c}{(dp/dT_c)^2}. \tag{11}$$

From (11), with consideration of (10) and (8), we will obtain

$$\left(\frac{\alpha^2 K}{V} + \alpha \frac{dK}{dT_c} + K \frac{d\alpha}{dT_c} \right)^{-1} > \frac{2T_c}{p_0 \alpha K^2}. \tag{12}$$

Let us substitute (7) and (8) into (12):

$$\begin{aligned} &\left(\frac{\alpha^2}{V} \left(\frac{C_1 z^{5/3}}{\Omega_0 (1 + \alpha \delta T)^{5/3}} + \frac{C_2 z^2 r_0^2 \left(1 + \frac{\alpha}{3} \delta T\right)^2}{\Omega_0^2 (1 + \alpha \delta T)^2} \right) + \right. \\ &+ \left(C_1 \left(\frac{z}{\Omega} \right)^{5/3} \frac{\delta T}{1 + \alpha \delta T} + \frac{4 C_2 z^2 r_0^2}{3 \Omega_0^2} \frac{1}{(1 + \alpha \delta T)^2} \right) \frac{d\alpha}{dT} + \\ &\left. + \left(\frac{C_1 z^{5/3}}{\Omega_0 (1 + \alpha \delta T)^{5/3}} + \frac{C_2 z^2 r_0^2 \left(1 + \frac{\alpha}{3} \delta T\right)^2}{\Omega_0^2 (1 + \alpha \delta T)^2} \right) \frac{d\alpha}{dT} \right)^{-1} > \frac{2T_c}{p_0 \alpha K^2}. \end{aligned} \tag{13}$$

Having denoted $C_1 \left(\frac{z^{5/3}}{\Omega_0} \right) \frac{1}{(1 + \alpha \delta T)^{8/3}} = \varphi_1$, $C_2 \left(\frac{z r_0}{\Omega_0} \right)^2 \frac{1}{(1 + \alpha \delta T)^2} = \varphi_2$, we will rewrite (13) in more compact form

$$\begin{aligned} &\left(\frac{\alpha^2}{V} \left(\varphi_1 + \varphi_2 \left(1 + \frac{\alpha}{3} \delta T \right)^2 \right) + \alpha \left(\varphi_1 \delta T - \frac{4}{3} \varphi_2 \right) \frac{d\alpha}{dT_c} + \right. \\ &\left. + \left(\varphi_1 + \varphi_2 \left(1 + \frac{\alpha}{3} \delta T \right)^2 \right) \frac{d\alpha}{dT_c} \right)^{-1} > \frac{2T_c}{p_0 \alpha \left(\varphi_1 + \varphi_2 \left(1 + \frac{\alpha}{3} \delta T \right)^2 \right)}. \end{aligned} \tag{14}$$

Inequality (14) in principle allows us to predict the possibility for the existence of rarefaction shock wave in a material on the basis of the known values of $d\alpha/dT$ near the point of phase transition and to determine the rate of change from the temperature of the coefficient of thermal expansion.

The hypothesis for the existence of rarefaction shock waves associated with phase transition of the IInd kind in solid bodies allows us from the viewpoint of several different positions to deal with the problem of the formation of so-called white phases which arrive under conditions of high-speed loads. The experimental verification of these results is the subject of further research.

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